Interaction between ice sheet models and the solid Earth

Eveline van der Linden
Utrecht University
Institute for Marine and Atmospheric research Utrecht

May 31, 2010

Introduction There are two fundamentally different approaches to study the effect of changing ice loads on the Earth. The glaciological approach is to force an ice sheet model by changes in the climate and calculate the response of the Earth. The geophysical approach is to use uplift data as constraint to reconstruct the loading history of the Earth. The first approach fulfills the ice dynamical constraints but those not necessarily fulfill the observations in uplift data. For the second approach this is the other way around.

To bring the two approaches closer together, we try to improve the treatment of isostatic adjustment in the ice sheet model. In order to do so we compare the flexural Earth model often used by glaciologists to a more sophisticated self-gravitating viscoelastic spherical Earth model.

The aims were, in short:

1. Step 1: Literature study of relevant papers; Study of theory of the earth response.
2. Step 2: Identify the differences in present-day uplift from the 3D ice model and ICE4/5G results.
3. Step 3: Implementation of a physical sound Earth model.

Model description In the present ice sheet model a flexural model represents the response of the Earth to ice loads. In the flexural model the theory of elastic plates is used. The time-dependent adjustment of the bedrock takes place exponentially, with τ the relaxation time in which the situation has adapted a factor e.

In the self-gravitating viscoelastic spherical Earth model (SGVE) less simplifications are made and therefore it is thought to be more realistic than the flexural model. The main assumption in the SGVE model is that the Earth is spherically layered with a radial density distribution. As a consequence, the problems that can be solved by the SGVE model are one-dimensional. Furthermore, the Earth is assumed to be a perfectly incompressible and non-rotating body. The response of the Earth consists of changes in the displacement field and in the gravity field.

Experiment 1: Estimating realistic bedrock parameters from the SGVE model We considered a steady state experiment in which the ice model is forced with the present day monthly mean surface temperature and precipitation pattern as prescribed by ERA-40, with an temperature offset relative to present day of -10°C.

To see whether it is useful to replace the treatment of isostatic adjustment in the ice model with the SGVE model, we made a rough estimate of some typical bedrock parameters using the outcome of the SGVE model. The particular bedrock parameters are the relaxation time τ, the flexural rigidity D and the radius of relative stiffness L_r. Although τ and L_r are not defined in the SGVE model as physical parameters, we use them as mathematical constraints to approximate the shape of the bedrock deformation.

To obtain the values of the three bedrock parameters, we run the SGVE model and compared the bedrock deformation profile with the profile of deformation of the flexural model. The density and rheological structure of the Earth model are taken from the Preliminary Reference Earth Model (PREM) and averaged over the thickness of the layers. All of the densities and shear moduli are PREM-averaged, except the densities of the two visocoelastic mantle layers, which have the PREM values just below and just above the 670 km depth discontinuity.

The flexural rigidity D is calculated from Young's modulus E and Poisson's ratio. To obtain the relaxation time τ and radius of relative stiffness L_r, we inserted a single disc load instantaneously on the Earths surface at time $t = 0$. Because the SGVE model is spherically symmetric, the resulting deformation of the Earth does not depend on its specific location and is representative for a disc load placed on any location of the Earth. The mass of the disc load in the SGVE model should be taken equal to the mass of the ice load in one grid box of the ice sheet model. We choose to use a disc load instead of a rectangular load because it requires much less computation time, while only minor errors are introduced in the bedrock deformation pattern. The radial displacement profiles and time histories for an ice height $H_i = 1000$ m and different values of T_e are computed. The ice height does not influence the shape or time evolution of the bedrock profile, but only influences its magnitude. So L_r and τ are independent of H_i.

For the lithospheric thicknesses T_e =70 km and T_e =100 km the general shape of the deformation is rather similar to the Kelvin function. Thus, for these values we can keep for a rough approximation the Kelvin function and tune its amplitude and shape with the outcome of the SGVE model. In our approximation, we define the radius of lithospheric rigidity L_r as the radius from the centre of the load at which the bedrock depression is equal to zero, divided by four. The radius of rigidity increases with increasing elastic lithospheric thickness. The relaxation time τ can be estimated from the time histories and is equal to the e-folding time of the bedrock deformation beneath the centre of the load. This is done by taking the time at which about 63% of the steady state bedrock deformation is completed. We observe that for a thicker lithosphere, the relaxation time is smaller.

Experiment 2: Introducing estimated bedrock parameters into the ice model The default values of the parameters D and τ in the ice model are $D = 4.0 * 10^{22}$ Nm and $\tau = 3000$ yr. The corresponding radius of rigidity L_r is $3.3*10^4$ m. From the results of Experiment 1, it turns out that the isostatic adjustment of the SGVE model is different than the flexural bedrock adjustment in the ice model. The small flexural rigidity and radius of rigidity correspond with a very small lithospheric thickness in the SGVE model, whereas the small relaxation time corresponds to a thick lithosphere in the SGVE model. This contradicting behaviour confirms that the isostatic adjustment process in the ice sheet model is rather different than the adjustment in the SGVE model.

Conclusion We did an extensive literature study about the theory of the Earth response and the SGVE model (Step 1). Although we did not yet completely identify the differences in present-day uplift from the three-dimensional ice model and ICE4/5G results, we made some important steps forward in this direction (Step 2). We can conclude that we have made significant progression towards the development of a physical sound Earth model (Step 3).

Ongoing research We have shown that the isostatic adjustment in the ice sheet model is different than in the SGVE model, which is thought to be more realistic. Therefore, we will try to replace the bedrock treatment in the ice sheet model by the SGVE model. In order to do this, we will couple the SGVE model to the ice sheet model. This is done by letting the programs communicate interactively as follows. Firstly, a table with ice thicknesses is calculated with the ice sheet model for $t = t_0$. This ice thickness table is introduced into the SGVE model, after which the SGVE model computes the radial displacement field of the Earth. The radial displacement field is interpolated on the rectangular ice sheet grid en introduced in the ice sheet model. Then the ice sheet model computes the new bedrock profile and calculates the new ice thickness field for $t = t_1$, etc. In this way, the ice sheet model is able to account for the response of the Earth as obtained with the SGVE model.